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Study the Behavior of Total Least Squares Technique in Adjusting GPS Field Data - A Case Study

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Abstract The main research fields in mathematical and satellite geodesy are computations and adjustment to assess the magnitude of errors and to study their distributions in terms of when they are within the adequate tolerance. In order to achieve this study's objective, a GPS field direct results were adjusted using Least Squares (LS) and Total Least Squares (TLS) techniques. The difference between the LS and TLS, is that the first recognizes errors only in the observation matrix, adjusting observations in order to get the sum of their squared residuals minimum, whereas the latter acknowledge errors in both the observation matrix and design matrix, which minimizes the noise in both matrices. We used two case studies in this research, the first case study deals with baselines up to 30 km; and second one deals with baselines up to 4 km. The applied two solutions demonstrate that the result from LS technique is approximately the same of TLS on GPS network adjustment in some cases. This study main purpose is to compare the efficiency of the LS and TLS, assessing their individual accuracy and selecting the most effective method in adjusting GPS baselines. Based on statistical indicators of mean and root mean square error each model was assessed. After applying the LS and TLS techniques individually for the same data sets, it is noticed that, LS and TLS in the first case study gave root mean square error equal to 5.01mm and 5.12mm respectively. Again, in the second case study, both techniques gave the same results. Accordingly, this study highlights the efficiency of LS and TLS in solving different problems in satellite geodesy.

Keywords Field data, GPS networks, Adjustment, Least Squares, Total Least Squares

1. Introduction

Least Squares (LS) is the most commonly used adjustment method in geodesy. In the recent decades it has been applied in many geodetic areas. Notable among them; approximation of the surfaces in engineering structures [1], finding the relationship between global and Cartesian coordinates [2], predictions of local coordinates [3], converting GPS data from global coordinate system to the National coordinate system [4]. Also, GPS field data is adjusted using LS like any other geodetic observations.

Like any other surveying measurements, the field measurements contain errors and need to be adjusted. Adjustment is usually done using LS [5], [6], but the least squares which is based on regression analysis considers the errors in the observations matrix only [7]. As a matter of fact, errors do exist in both the design and observation matrices which must be put into consideration [5], [8], [9], [10], and [11] so, Total Least Squares technique (TLS) should be tested against LS technique.

The difference between the LS and TLS, is that the first acknowledges errors only in the observation matrix and adjust observations to make the sum of their squared residuals minimum. Whereas the latter considers errors in both the observation matrix and the design matrix, which minimizes the errors in both matrices to yield a better estimate. The models used in adjusting surveying GPS networks to present this study are LS and TLS. Some studies are carried out by using TLS technique, data and perturbation size [12]. [7], [5], [14], [6], [15] have applied TLS to solve many surveying problems.

2. Methodology

The GPS measurement method is divided into Baseline and Network model. In the Baseline method, one GPS Rover is fixed on the reference station permanently while the other rover is moving on the other ground stations to get the relative relationship between the reference station and the rest of the ground stations sequentially.

While the idea of networking depends on monitoring the observation between the reference station and the rest of the stations of the network simultaneously.

Differential post-processing can be divided into three segments; data analysis and validation, baseline processing,

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and network adjustment [16]. The data analysis and validation stage happen when the data loads into the processing software. It involves examining the data and removing any noisy data, cycle slips, and dependent baselines to improve the survey. Baseline processing is important to the post-processing method. For baseline observations, it carried out when put the GPS receiver at the base station B and the rover at the points J, K, A and L respectively. Baselines are formed by collecting carrier and phase data at two different points at the same time. By processing the baselines before adjustment, any outliers or bad points in the survey can be identified and isolated by analyzing the statistics that the baseline processor produces. The data can then be adjusted by conventional post-processing in a suitable software package which is Trimble Business Center (TBC).

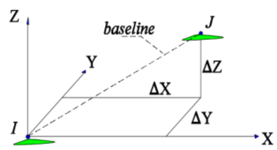


Figure 1-a. Vector Baseline

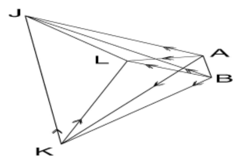


Figure 1-b. Network

Observation equation for baseline IJ:

$$X_J = X_I + \Delta X_{IJ} + \nu_{x_{IJ}}$$
$$Y_J = Y_I + \Delta Y_{IJ} + \nu_{y_{IJ}} \quad (1)$$
$$Z_J = Z_I + \Delta Z_{IJ} + \nu_{z_{IJ}}$$

Least Squares Adjustment

It is a statistical technique that is capable of determining the line of best fit of a model and seeks to find the minimum sum of the squares of residuals. This method is widely used in regression analysis and estimation [17]. Considering a system of equations in the form as denoted by Equation 2 to be solved by least squares.

$$AX \approx L \quad (2)$$

Where X is the vector of unknown parameters, the i-th row of the design matrix (or data Matrix) $\boldsymbol{A} \in \boldsymbol{R^m}$ and the vector of observations $\boldsymbol{L} \in \boldsymbol{R^n}$ contain the measurements of the variables a_1, …a_m and L, respectively [5], [18]. In the classical least squares LS technique the measurements A of the variables a_i (left hand side of Eq. 2) are assumed to be free of error and hence, all errors are confined to the observation vector L (the right hand side of Eq. 2), however, this assumption is frequently unrealistic sampling errors, human errors, modelling errors and instrumental errors. By solving the linear system of Eq. 2 (if m=1) with A =$[a_1, \ldots, a_n]^T$ and L= $[l_1, \ldots l_n]^T$.

The solution of the unknown parameters X by LS approach can be achieved as denoted by Equation 2:

$$X = [A^T A]^{-1} [A^T L] \quad (3)$$

The corresponding error vector V can be achieved by using Equation 4 as denoted by:

$$V = AX - L \quad (4)$$

Total Least Squares Adjustment

TLS is one method of fitting that is appropriate when there are errors in both the design matrix A and observation vector L. It amounts to fitting a best subspace to the measurement data (A^T_i, L_i), i=1,….n where A^T_i is the i-row of A [19]. In case of TLS which assumes that all the elements of the data are erroneous, the equation will be as follow:

$$(A + \Delta E)\, X = L + \Delta L \; (Rank(A) = m < n) \quad (5)$$

$$\left\| [A; L] - [\hat{A};\, \hat{L}] \right\|_F \; [\hat{A};\, \hat{L}] \in R\, n(m+1) \quad (6)$$

Where ΔL; is the error vector of observations, ΔE is the error matrix of design matrix A, m is the number of unknowns and n is the number of observations. The assumption that both have separately and identically distributed rows with zero mean and equal variance. The basic TLS problem can be solved using the Singular Value Decomposition (SVD). The SVD of the augmented matrix $[A; L]$ can be computed according to Eq. (7)

$$[A; L] = U \Sigma V^T \quad (7)$$

Where,

$U = [u1{,}1, \ldots, u1, n, \ldots, un, 1, \ldots.un, n] \in R\; n \times n$
$V = [v1{,}1, \ldots, v1, m+1, \ldots, vm, m+1, \ldots.vm+1, m + 1] \in R\; (m+1)(m+1)\; and$
$\Sigma = [\sigma 1{,}1, \ldots, \sigma 1, m+1, \ldots, \sigma m+1, m + 1, \ldots.\sigma n, 1, \ldots \sigma n, m+1] \in R\; n(m+1)$
$matrix\ with\ diagonal\ elements.$
$So\ the\ matrix\ \Sigma\ will\ be\ as$
$\Sigma = diag(\sigma 1, \ldots \sigma m, \sigma m + 1)$

The solution of TLS is obtained after the rank reduced from (m+1) to (m) for Eq. (6).

$$X = \frac{-1}{V_{m+1,m+1}} V_{m+1} \quad (8)$$

The biased-corrected variance component estimator is computed as in Eq. (9)

$$\sigma_0^2 = \frac{(L-AX)^T P\, (L-AX)}{r} \quad (9)$$

Where r denotes the redundancy and is equal to (m-n).

Theoretically, the TLS technique provides higher reliability as it acknowledges both sides of the noise contamination problem – pertaining design matrix and observation vector – whereas the LS method considers only the observation vector (neglecting design matrix).

This research main objective is to evaluate the performance of total least squares and least squares application on the adjustment of processed baseline. The true reference in this case is the network adjustment using traditional software packaging.

The methodology provided in this research as summarized in Figure (2) is described as follows.

- Processing all possible baselines of two case studies were applied, where baseline is the coordinate vector resulting from any station pair. The first case study is of long baselines that represents separation distances up to 30 Km. However the second one is of short base lines of separation distances up to 4 Km.

- Then the adjustment of processed baseline is done, where:
  - o The network adjustment using TBC software package is applied. This considered as the true reference.
  - o Solution (1): Adjusting the processed baselines using Least Squares (LS)
  - o Solution (2): Adjusting the processed baselines using Total Least Squares (TLS)
- The comparison between the two solutions and the true reference are achieved to yield the residuals between the calculated points coordinates
- The assessment of the accuracy is based on calculating Errors (E), Mean, and root mean square error (RMSE) of the results using Eq. (10), Eq. (11), and Eq. (12).

$$\mathbf{E_i} = \sqrt{\mathbf{\Delta X_i}^2 + \mathbf{\Delta Y_i}^2 + \mathbf{\Delta Z_i}^2} \quad (10)$$

Where $\Delta X_i, \Delta Y_i, \Delta Z_i$ are the residuals (the difference between the coordinates established from network adjustment and the computed one by LS and TLS) in X, Y and Z coordinates respectively.

$$\mathbf{Mean} = \frac{\sum_{i=1}^{N} \mathbf{E_i}}{n} \quad (11)$$

Where N is the number of stations.

$$\mathbf{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} \mathbf{E_i}^2}{n}} \quad (12)$$

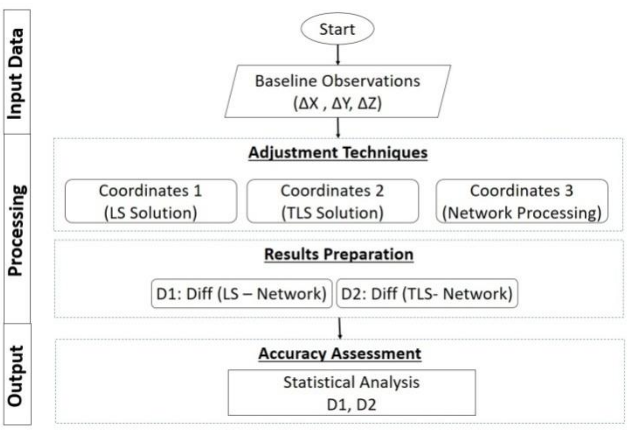
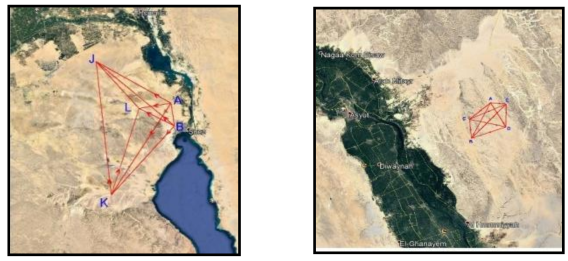


Figure 2. Block diagram of the proposed methodology

3. Study Area

The proposed methodology in this research is applied at two sites with different baseline ranges. Site 1 is located in Suez governorate - Egypt of area 300 km^2 with baseline range up to 30 km, Site 2, which is located in Asyut governorate – Egypt of area 6 km^2, has baseline separation up to 4 km. Figure (3) represents the location of the two sites.



(a) Site 1 (b) Site 2

Figure 3. Study area locations

The data used here is dual frequency for GPS; having them both operating in static mode where the user can get the results in ITRF reference frame. The datasets were collected on February 2017. In addition to these sites, a local station for each site is established and continuous observations were taken from this site at the same previous time span. The data is observed using receiver type TRIMBLE R8 and antenna type TRMR8_GNSS3. Each dataset consist of 5 stations including the base station. The GPS network is adjusted with fixing the base station only.

4. Results and Analysis

The methodology proposed is applied on the above-mentioned solutions. The coordinates are calculated with least squares adjustment, total least squares adjustment, and network adjustment. The residuals D1 and D2 are computed at each point. Where D1 represents $(\Delta X, \Delta Y, \Delta Z)$ the difference between the coordinates established from network adjustment and the computed one by Least Squares (LS), and D2 represents $(\Delta X, \Delta Y, \Delta Z)$ the difference between the coordinates established from network adjustment and the computed one by Total Least Squares (TLS). The mean of absolute residuals and root mean square error are computed for the different solutions. The results of the solutions are tabulated and discussed below. Table (1) represents the results for long baselines; while Table (2) denotes the results for short baselines.

It is obvious that the residuals of TLS for the majority of the points in X, Y, Z directions are almost equal to the residuals of LS adjustment.

Table 1. Results of Case Study (1) – Long baselines (up to 30km)

ID	D1 (LS - Network)			D2 (TLS - Network)			D1 - D2		
	ΔX (mm)	ΔY (mm)	ΔZ (mm)	ΔX (mm)	ΔY (mm)	ΔZ (mm)	ΔX (mm)	ΔY (mm)	ΔZ (mm)
J	-4.47	0.35	-3.17	-4.773	0.338	-3.188	0.303	0.012	0.018
L	0.774	-1.411	0.104	0.754	-1.48	0.098	0.02	0.069	0.006
K	-2.66	0.246	-1.552	-2.67	0.24	-1.5775	0.01	0.006	0.0255
A	0.358	-2.578	0.495	0.35	-2.591	0.481	0.008	0.013	0.014

The resultant (E) of the residuals in mm for each case are computed as presented in Figure (4). Also, the mean error as well as RMSE are calculated. Table (2) illustrates the values of the results.

Table 2. Accuracy Assessment results of Case Study (1) – Long baseline (up to 30km)

ID	E (LS) mm	E (TLS) mm
J	5.5	5.7
L	1.6	1.7
K	3.1	3.1
A	2.6	2.7
Min	1.6	1.7
Max.	5.5	5.7
Mean	3.2	3.3
RMSE	3.5	3.6

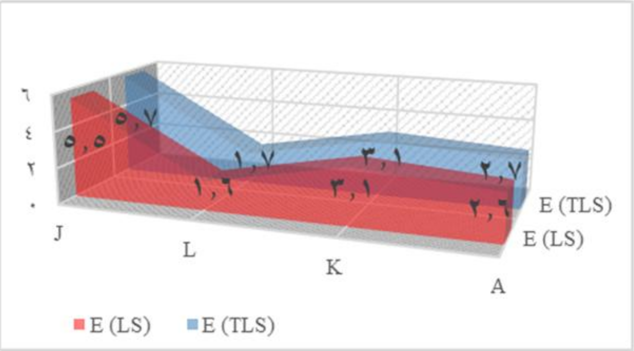


Figure 4. Computational accuracy comparison of LS, TLS (Case Study 1)

Similarly, Table (3) illustrates the results for short baselines case study.

Table 3. Results of Case Study (2) – Short baselines (up to 4Km)

ID	D1 (LS - Network)			D2 (TLS - Network)			D1 - D2		
	ΔX (mm)	ΔY (mm)	ΔZ (mm)	ΔX (mm)	ΔY (mm)	ΔZ (mm)	ΔX (mm)	ΔY (mm)	ΔZ (mm)
A	-0.101	0.264	-0.790	-0.100	0.260	-0.700	-0.001	0.004	-0.090
B	0.540	-0.162	0.482	0.543	-0.160	0.480	-0.003	-0.002	0.002
C	-1.088	0.767	-0.360	-1.080	0.760	-0.300	-0.008	0.007	-0.060
D	0.505	-0.863	0.667	0.500	-0.860	0.660	0.005	-0.003	0.007

There is no difference between the residuals of TLS and its corresponding values in LS for the whole points in X, Y, Z directions.

Table 4. Accuracy Assessment results of Case Study (2) – Short baselines (up to 4km)

ID	E (LS) mm	E (TLS) mm
A	0.839	0.753
B	0.742	0.742
C	1.379	1.354
D	1.202	1.194
Min	0.742	0.742
Max.	1.379	1.354
Mean	1.040	1.011
RMSE	1.072	1.046

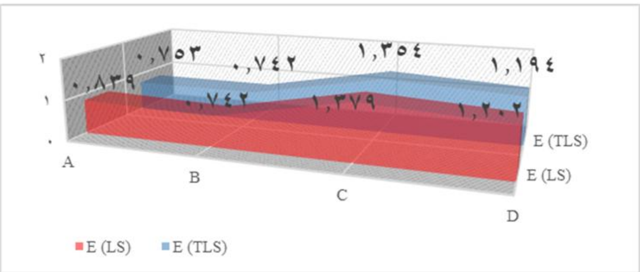


Figure 5. Computational accuracy comparison of LS, TLS (Case Study 2)

Respectively, Table (4) illustrates the values of the resultant residuals in mm for each case as shown in Figure (5). Furthermore, the mean error and RMSE are also presented.

In short baselines, the error values are identical, this lead to a conclusion that TLS adjustment has no better effect for short base lines.

5. Conclusions

The research main objective is to examine the performance of total least squares with reference to least squares for both case studies to adjust the GPS networks. Through the results of the applied solutions, it can be concluded that the two techniques gave approximately the same results in two cases. The mean and RMSE of LS in the case study 1 are 4.38mm and 5.01mm respectively. The mean and RMSE of TLS in the case study 1 are 4.46m and 5.12mm respectively. Finally, the TLS technique give the same result of LS technique.

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